

11-695: AI Engineering

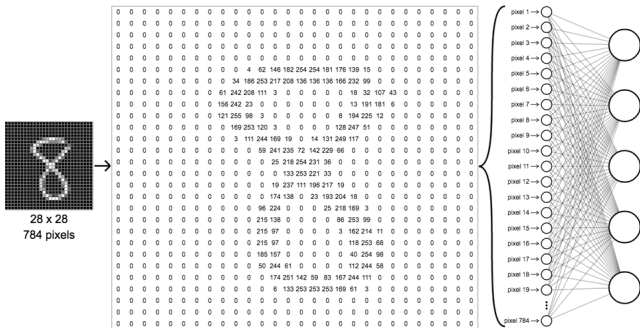
Recurrent Neural Networks

LTI/SCS

Spring 2020

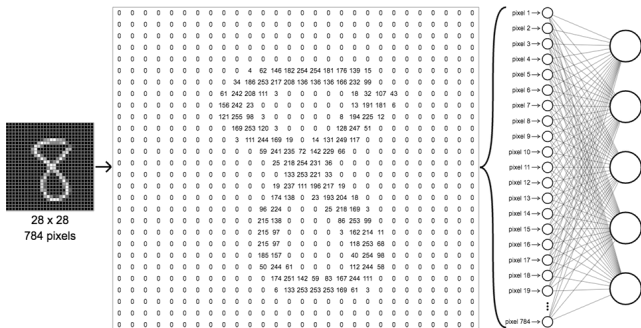
- 1 Motivation
- 2 RNNs as Functions Composition
- 3 Modelling RNNs
- 4 Extensions

MLP a one-size-fits-all approximator?

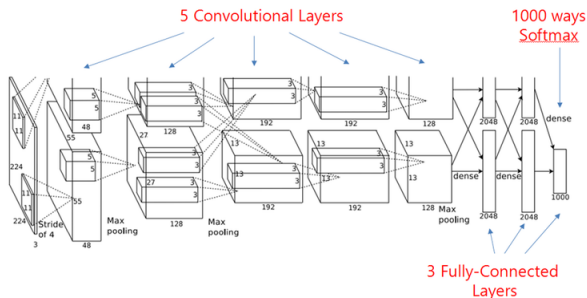


- A general solution to general data

MLP a one-size-fits-all approximator?

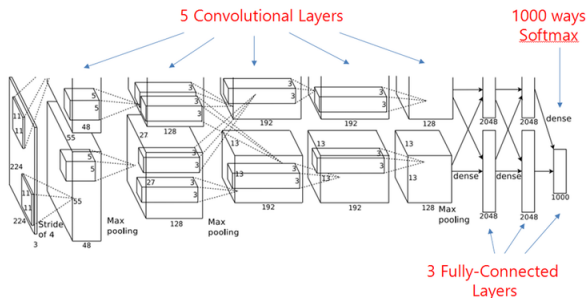


- A general solution to general data
- It has disadvantages: params, relations, ...



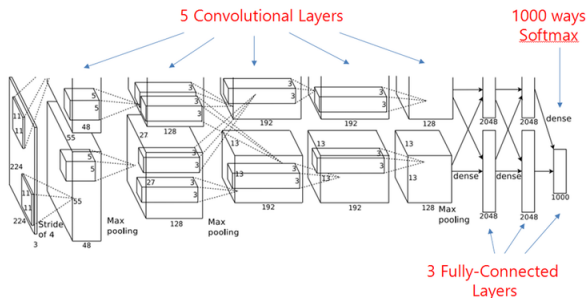
- It has a huge advantage in vision and related data

¹Not counting some special cases such as Fully CNN



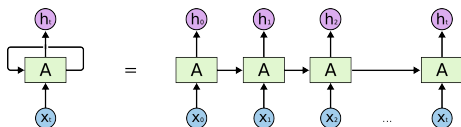
- It has a huge advantage in vision and related data
- But also normally requires fixed size of input¹

¹Not counting some special cases such as Fully CNN

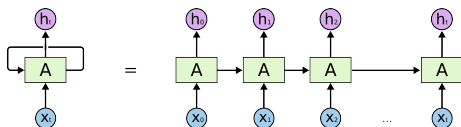


- It has a huge advantage in vision and related data
- But also normally requires fixed size of input¹
- With MLP, they are feed-forward type without “memory”

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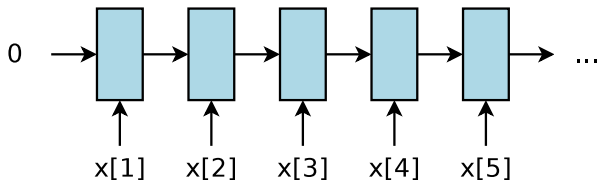


- Texts, video, trading, ...
- Treating samples independently is losing information
- We need a special model for time-series data
- Markov, Graphical Model, ... and

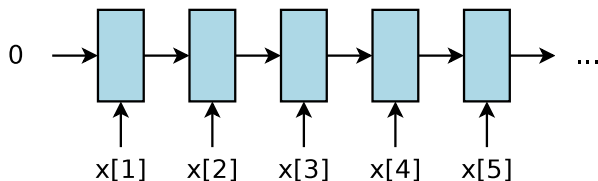


- Texts, video, trading, ...
- Treating samples independently is losing information
- We need a special model for time-series data
- Markov, Graphical Model, ... and
- Recurrent NN

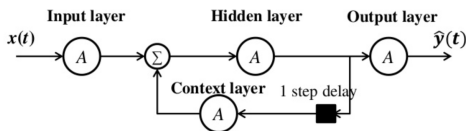
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- Processes a sequence of signals
 - $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_T \in \mathbb{R}^D$
- ... in a sequential order
 - $\mathbf{h}_0 = \mathbf{0}_H$
 - $\mathbf{h}_t = \mathbf{f}(\mathbf{x}_t, \mathbf{h}_{t-1})$
- Designing an RNN primarily means modelling sequential representation with a function $\mathbf{f} : \mathbb{R}^D \times \mathbb{R}^H \rightarrow \mathbb{R}^H$

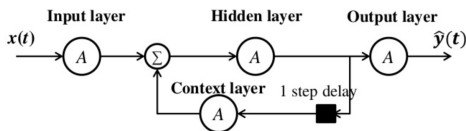


- With the function $f(\mathbf{x}, \mathbf{h}) = \mathbf{x} + \mathbf{h}$
 - $\mathbf{h}_0 = \mathbf{0}_H$
 - $\mathbf{h}_t = \mathbf{h}_{t-1} + \mathbf{x}_t$
- This network requires $D = H$, and is really naive ...



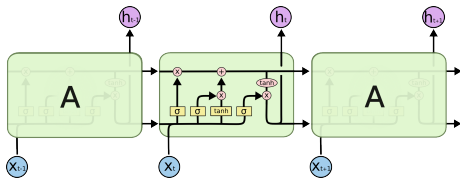
- With the function $\mathbf{f}(\mathbf{x}, \mathbf{h}) = g(\mathbf{x} \cdot \mathbf{W}_{ih} + \mathbf{h} \cdot \mathbf{W}_{hh} + \mathbf{b}_{hh})$
 - g is an activation function, e.g. tanh, sigmoid, ...
 - $\mathbf{W}_{ih} \in \mathbb{R}^{D \times H}$, $\mathbf{W}_{hh} \in \mathbb{R}^{H \times H}$ are the *shared* parameters.
- Much efficient, invented in 1990, got popular in 2011.

²https://onlinelibrary.wiley.com/doi/epdf/10.1207/s15516709cog1402_1



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- Vanishing gradients problem

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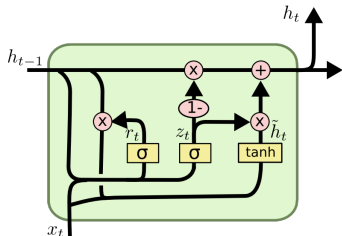
- Invented in 1997 and got super popular since 2014.

$$\begin{bmatrix} \mathbf{i} \\ \mathbf{f} \\ \mathbf{o} \\ \mathbf{g} \end{bmatrix}^T = \begin{bmatrix} \text{sigmoid} \\ \text{tanh} \\ \text{sigmoid} \\ \text{sigmoid} \end{bmatrix} \mathbf{W}_{H \times (D+H)} \cdot \begin{bmatrix} \mathbf{x}_t^T \\ \mathbf{h}_t^T \end{bmatrix} \quad (1)$$

$$\mathbf{c}_t = \mathbf{i} \otimes \mathbf{g} + \mathbf{f} \cdot \mathbf{c}_{t-1}$$

$$\mathbf{h}_t = \mathbf{o} \otimes \tanh \mathbf{c}_t$$

³<https://www.bioinf.jku.at/publications/older/2604.pdf>



- Combines forget and input gates to change \mathbf{f}

$$\mathbf{z} = \text{sigmoid}(\mathbf{x}_t \cdot \mathbf{W}_{xz} + \mathbf{h}_{t-1} \cdot \mathbf{W}_{hz})$$

$$\mathbf{r} = \text{sigmoid}(\mathbf{x}_t \cdot \mathbf{W}_{xr} + \mathbf{h}_{t-1} \cdot \mathbf{W}_{hr})$$

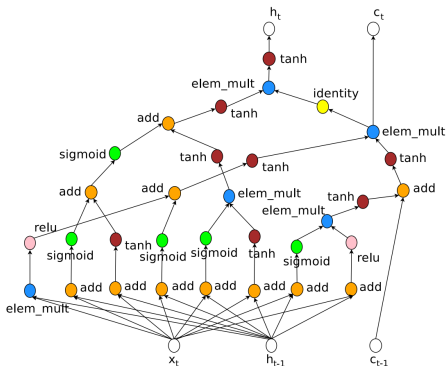
$$\tilde{\mathbf{h}} = \text{sigmoid}(\mathbf{x}_t \cdot \tilde{\mathbf{W}}_x + (\mathbf{r} \cdot \mathbf{h}_{t-1}) \cdot \tilde{\mathbf{W}}_h)$$

$$\mathbf{h}_t = (1 - \mathbf{z}) \otimes \mathbf{h}_{t-1} + \mathbf{z} \otimes \tilde{\mathbf{h}}$$

(2)

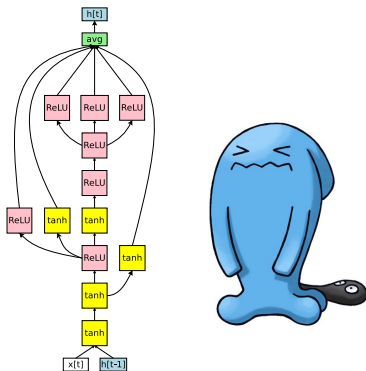
- Cell state is same as hidden's (output), unlike LSTM, so simpler

⁴<https://arxiv.org/pdf/1409.1259.pdf>



- You can also use a computer to generate good formulas for f

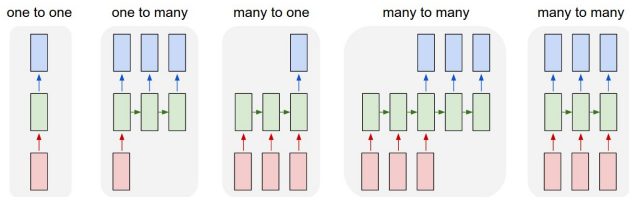
⁵<https://arxiv.org/pdf/1611.01578.pdf>



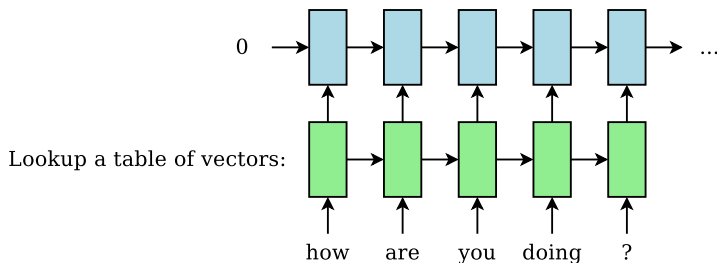
- Yet another one, also generated by a computer

⁶<https://arxiv.org/pdf/1802.03268.pdf>

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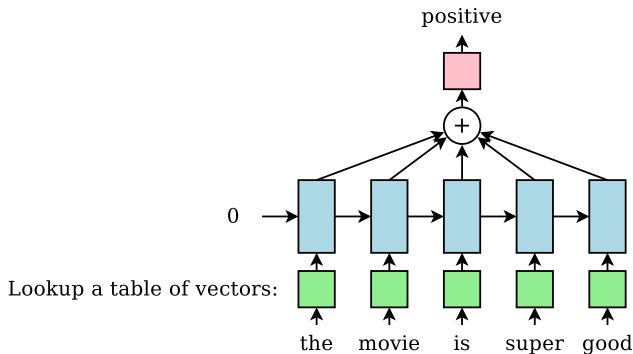


- You can be model many layouts of input and outputs
- You can be *very* creative about RNNs:
 - How to choose the input sequence $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_T$
 - What to do with the “hidden” sequence $\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_T$

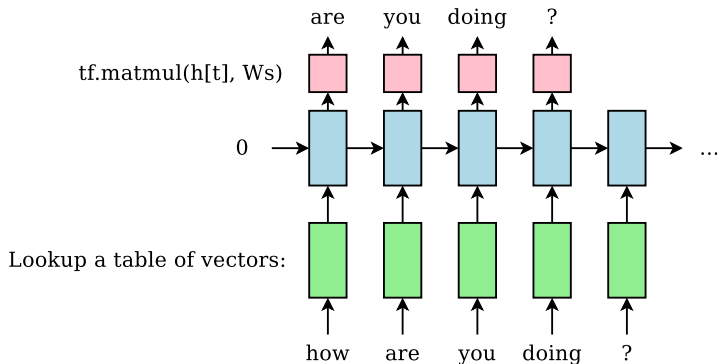


- To process a sequence of words
 - Store a dictionary that maps words to vectors in \mathbf{R}^D
 - Use these \mathbb{R}^D vectors as inputs to an RNN.
- Each word is embedded to a lower dimensional space

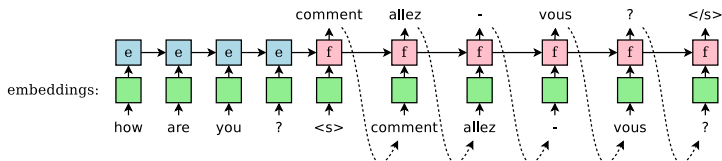
⁷<http://jmlr.org/papers/volume3/bengio03a/bengio03a.pdf>



- You can sum over the \mathbf{h}_t and hook up a softmax head to make a prediction about your sequence
 - Example: sentiment analysis

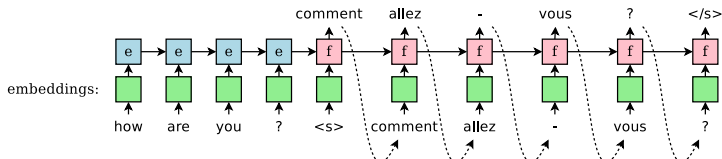


- You can use the \mathbf{h}_t as to predict the next word in your sequence
 - Example: *language model*
 - Because it can model $p(w_t|w_{<t})$



- Only use the softmax heads where you want to generate a translated word
 - It's called *neural machine translation*
 - Because it can model $p(\mathbf{t}_t | \mathbf{t}_{<t}, \mathbf{s})$
- A **very important model** to remember!
- Can be extended to many other problems in practice such as image captioning, image synthesis, multimodal models, ...

⁸<https://arxiv.org/pdf/1409.3215.pdf>



- Yet another way to manipulate your \mathbf{h}_t states.
- \mathbf{e}_i , \mathbf{f}_j are your blue and red states

$$\alpha_{j,i} = g(\mathbf{f}_j, \mathbf{e}_i)$$

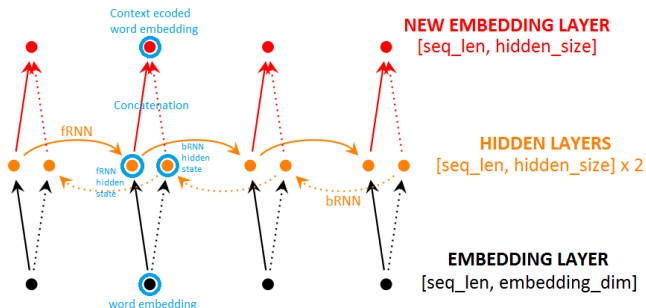
$$a_{j,i} = \text{Softmax}(\alpha_{j,1}, \alpha_{j,2}, \dots, \alpha_{j,|s|})$$

$$c_j = \sum_{i=1}^{|s|} a_{j,i} \mathbf{e}_i$$

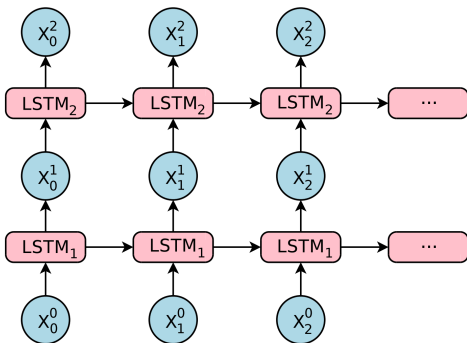
(3)

⁹<https://arxiv.org/pdf/1409.0473.pdf>

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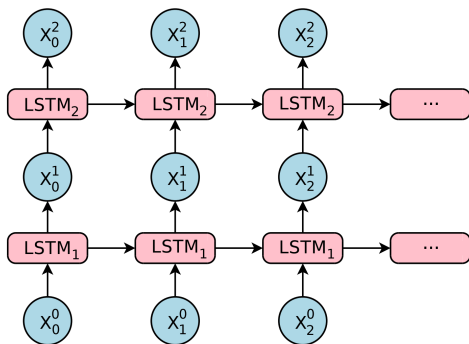


- Model 2-way relations between time steps: past & future
- Actually contain 2 RNNs with opposite directions
- Usually concatenate the hidden states of the two



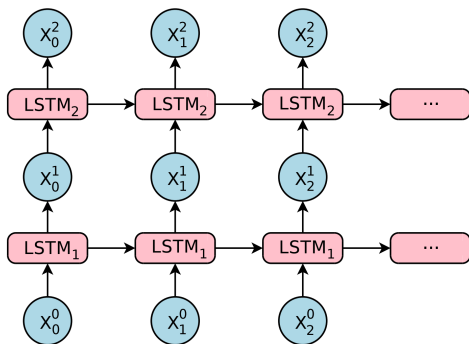
- Simply stacking layers on top of each other

¹⁰<https://arxiv.org/pdf/1409.3215.pdf>



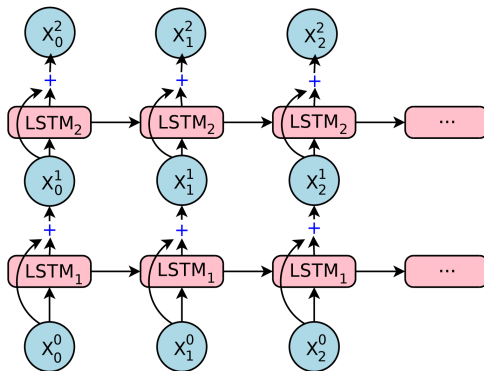
- Simply stacking layers on top of each other
- Harder to train, but found better than single-layer one¹⁰

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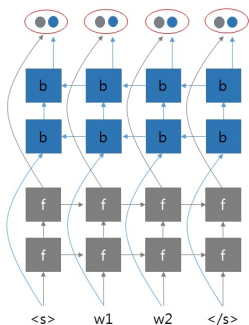
- Simply stacking layers on top of each other
- Harder to train, but found better than single-layer one¹⁰
- Gradients exploding or vanishing problem

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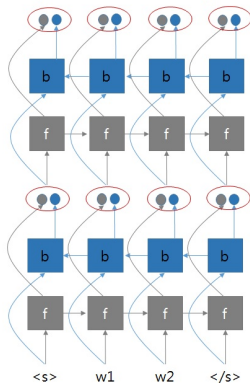


- Address gradient exploding or vanishing

2-layer bidirectional LSTM : type 1



2-layer bidirectional LSTM: type 2



- Can combine deep with bidirectional
- Can vary with many options